

Analysis and Improvement for Image Encryption Algorithm Based on Multiple Chaotic Mapping

Xixun Liu*

College of Information Engineering, Shaanxi Polytechnic Institute, Xianyang, 712000, P.R. China

Abstract: In order to enhance the security of chaotic encryption, a dynamic encryption algorithm based on chaotic discrete map. The algorithm USES two chaotic cascade subsystems. Subsystem consists of simple chaotic mapping, its output with the addition of plaintext modulus after generating cipher text, and subsystem number of iterations is dynamic change, increase the unpredictability of cipher text. Simulation and safety analysis show that the algorithm of the key space is large, is sensitive to plaintext and key, can effectively resist by difference features and statistical characteristics, the phase space reconstruction.

Keywords: Chaos, chaos map, encryption, secure communications.

1. INTRODUCTION

In nearly a decade, with the rapid development of information network technology, multimedia technology in various fields of application is changing. Digital image has become one of the main interaction object Internet undoubtedly. Digital image in military systems, government agencies, medical systems, defense systems and the financial system has been widely applied in such important institutions; it also means that the image in the process of transmission exist great potential safety hazard [1]. If at the time of transmission by a third party malicious interception, tampering, illegal copy at random, arbitrary transmission, the consequences will be very serious. So the safety of the digital image transmission problem is paid great attention to by the public [2].

Over the past decade, with the rapid development of information network technology, multimedia technology in various fields can be described with each passing day. Digital image has undoubtedly become one of the main objects of the Internet to interact. Digital images have been widely used in military systems important institutions, government agencies, medical systems, defense systems and financial systems, which also mean that the image there is a huge security risk in the process of transmission. If the transmission time by a third party malicious interception, tampering, unauthorized copying, any dissemination, the consequences will be very serious. So the security of digital image transmission problems is the widespread attention of the public. Therefore, information security has become a focus of concern, is today's hot and difficult research.

Chaos phenomenon is a kind of produced by a deterministic system, is sensitive to initial value extremely, similar to a random process, with a kind of noise, such as wide

frequency spectrum feature [2]. The chaos in the application of information encryption is one of current hot research topic. Literature, this paper proposes a image encryption algorithm based on multiple chaotic continuous dynamic system. In this paper, on the basis of this, using multiple chaotic mapping, design a kind of information encryption algorithm. The algorithm adopts the discrete chaotic mapping; do not need to solve differential equations. At the same time, the chaotic map and the dynamic changes of the number, improves the complexity and unpredictability of cipher text [3, 4]. The simulation shows that the sensitivity can effectively resist by difference features and statistical characteristics, the phase space reconstruction.

2. BASIC KNOWLEDGE OF THE DIGITAL IMAGE

The image is a kind of objective object similarity, vivid description or photo, is the most commonly used information carrier in the human social activities [5]. Objective object or the image is a representation; it contains information about the described object. It is the main source of information. According to statistics, we can get information about 75% from the visual. Using two-dimensional function $f(x, y)$ defines image, namely the x, y is the space coordinates, $f(x, y)$ is the point (x, y) amplitude, each point has a specific position (x, y) and the amplitude of $f(x, y)$, can be called pixels [5]. The gray image is a two-dimensional gray or brightness function. Color image is made up of three two-dimensional gray (RGB or HSV) functions, as shown in (Figs. 1 and 2):

Two-dimensional array composed of pixels, you can use A two-dimensional matrix $[m, n]$ said. M and n said width and height of the image and the value of the matrix element in a (I, j) , image in the case of line, i said the first j column of the grey value of pixels; i, j said geometric location.

As digital image more vivid than words or sounds, images and intuitive, contains very rich information, the digital image information security so the more people pay attention. Therefore this paper a digital image, based on research of



Fig. (1). Original image.

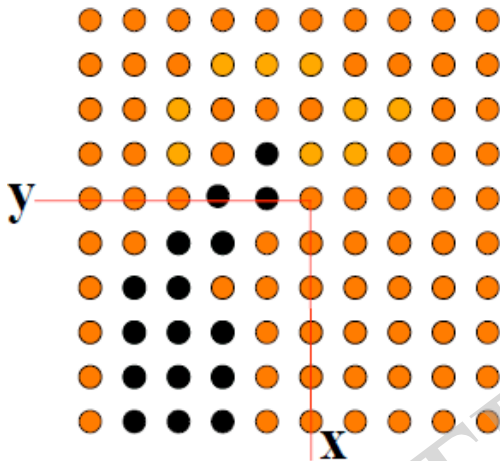


Fig. (2). The corresponding pixels.

digital image information security encryption technology [5]. The purpose of this task is to analyze the structure and characteristics of a digital image, and then encrypt and decrypt digital images, namely: the use of a certain image encryption algorithm to encrypt a pair of images, in order to achieve the purpose of hiding the original image, and then decrypt re-

store the original image. Image encryption algorithm has made abundant research results, people have devised a number of image encryption algorithm.

For gray image, pixel brightness can be a number between 0 and 255. Where 0 means black, 255 said white, other numerical between black and white and gray, as shown in (Figs. 3 and 4).



Fig. (3). Gray image.

Color images can use red, green, and blue triples two-dimensional matrix. Usually, triples every value is between 0 and 255, 0 indicates the corresponding color is not in the pixels, and represents the corresponding colors in the 255 pixels to obtain the maximum value [6].

Image is commonly used in computer storage formats: BMP, TIFF, EPS, JPEG, GIF, the PSD and PDF format. There are four basic types image in matlab, the index image, gray image, the RGB image, binary image [6]. Index images include a matrix color Map and data matrix X, gray image is composed of a certain range of color grey value data matrix, RGB image, stored in the mat lab for $m * n * 3$ data matrix, element defines each pixel of the color of the R, G, B value, binary image needs only a data matrix, take two each pixel gray value.

	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256
1	116	142	174	159	136	149	121	36	29	25	23	52	110	113	111	109	109	10
2	100	159	182	147	139	133	100	32	28	26	23	56	111	111	111	111	108	10
3	89	142	187	148	122	153	119	34	29	23	23	61	111	110	108	109	108	9
4	90	144	182	153	104	134	133	36	29	24	25	60	113	109	108	109	107	9
5	124	154	174	155	100	125	106	32	25	24	24	72	114	108	107	109	107	9
6	102	148	188	147	99	157	120	33	28	24	25	80	116	107	106	109	106	9
7	64	152	167	140	93	119	113	30	30	23	28	86	115	107	106	107	106	9
8	67	161	170	138	103	121	86	29	27	24	31	94	113	107	108	108	106	9
9	78	159	165	135	112	164	102	31	26	25	34	96	113	106	108	109	104	9
10	105	161	164	127	99	122	91	27	27	23	38	101	110	104	109	112	104	9
11	99	157	163	127	99	83	58	26	29	25	43	105	109	104	109	112	105	9
12	82	162	159	120	110	137	60	31	25	25	49	108	107	106	111	111	103	8
13	83	171	156	120	95	148	80	29	28	25	52	109	105	109	114	110	103	8
14	96	177	158	129	83	90	64	24	26	23	58	109	104	110	114	111	102	9
15	119	167	155	133	105	86	45	26	26	24	61	109	104	112	116	113	104	9
16	122	164	148	118	117	144	63	27	25	23	70	108	104	114	116	110	102	9
17	107	165	141	107	84	116	60	25	26	25	74	108	106	117	115	107	103	9
18	108	163	136	100	64	60	37	25	25	26	81	111	108	114	113	109	104	9
19	121	159	134	93	96	96	32	26	24	29	89	112	108	111	111	109	103	9
20	130	158	136	99	109	132	42	29	24	34	93	107	106	112	112	111	104	8
21	132	152	134	104	77	83	36	27	26	37	94	106	108	115	117	113	105	8
22	129	160	130	111	104	61	30	27	25	42	100	109	114	120	121	112	105	8
23	125	167	133	109	131	108	34	29	23	48	103	113	118	129	120	113	102	8
24	130	159	132	95	95	95	32	30	25	52	104	113	119	122	119	115	101	8
25	139	151	132	89	74	49	30	28	26	55	106	113	119	122	121	116	100	8
26	143	148	129	91	110	64	30	28	25	60	110	118	122	129	123	115	95	8
27	142	148	125	83	122	87	29	30	25	63	111	120	123	124	125	111	94	8
28	145	143	111	97	96	59	29	28	27	66	111	118	120	122	121	104	98	9
29	147	139	101	106	89	40	29	36	29	69	114	119	119	119	114	102	97	9

Fig. (4). Corresponding pixel gray value.

3. USING OF MULTIPLE CHAOTIC MAPPING INFORMATION ENCRYPTION ALGORITHM

3.1. Logistic Mapping

Logistic mapping is an autonomous one dimensional mapping [7]:

$$x_{n+1} = 1 - \mu x_n^2 \quad -1 \leq x_n \leq 1 \quad (1)$$

When $\mu = 2$, the system for the chaos with mapping. In this paper, we use Logistic mapping $\mu = 2$.

3.2. Cubic Mapping

$$x_{n+1} = rx_n^3 + (1-r)x_n \quad (2)$$

When $3.2 \leq r \leq 4$, the output for the chaotic sequence ($-1 \leq x_n \leq 1$). In this paper, Cubic maps are related to input the key value r .

3.3. Arnold Cat Mapping

Arnold Cat mapping is a confirmed area of chaotic mapping [8] :

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \end{bmatrix} \text{mod } 1 \quad (3)$$

The map while not attractor, but it increased use of matrix multiplication x and y to realize the "stretch", and through the modulus "fold" in the x and y , and stretching and folding is the chaotic motions of the two typical factors, Make its output for the chaotic sequence ($0 \leq x_n \leq 1, 0 \leq y_n \leq 1$). The Lyapunov index of (4):

$$l_1 = \ln \left[3 + \sqrt{5} / 2 \right] > 0 \quad (4)$$

The proposed encryption algorithm using Logistic and Cubic mapping constitute a and b two cascade chaotic subsystem, and based on the output of the cipher text feedback and Arnold Cat mapping changes a and b two sub-systems of the number of iterations, the output of the two subsystems and cipher text expressly after encryption function treatment [7].

4. BASED ON THE ANALYSIS OF THE CHAOTIC MAPPING INFORMATION ENCRYPTION ALGORITHM

The proposed chaos encryption system block diagram as shown. It's a and b by chaotic cascade subsystem, Arnold Cat mapping and encryption function $f(i)$. Subsystem of a and b are made by two levels: the discrete chaotic mapping connection subsystem is a 1 and 2 of Logistic mapping and Cubic; Subsystem b 1 and 2 are in turn is Cubic mapping and Logistic mapping [8] (Fig. 5).

In the case of a clear text m encryption, a and b each subsystem iterative η_i , after each time i output the U_i and w_i respectively. U_i, w_i and expressly m after encryption function $f(i)$ treatment, produce cipher text e_i .

Encryption system in the initial value of chaotic mapping and the initial number of iterations are associated with a key. Into three parts of K_1, K_2 and K_3 key K , which real $K_1, K_2 \in [3.2, 4], K_3$ is composed of n ($n-16$) character string ($K_3 = k_1, k_2, \dots, k_n$). The subsystem Cubic mapping parameters in a and b r respectively set up for the K_1 and K_2 . The rest of the initial value of chaotic mapping and according is K_3 and initial iteration times [9]. Encryption function:

$$f(u, w, m) = (L1000_{u,j} + L1000_{w,j} + m) \text{mod } 256 \quad (5)$$

4.1. Encryption Algorithm Steps:

- 1) according to the key $k_1 k_2 \dots k_n$ (binary) to generate the initial value:

$$\text{sum} = K_1 + K_2 + \dots + K_n \quad (6)$$

$$r = K_1 \quad K_2 \quad \dots \quad K_n \quad (7)$$

Subsystem of a initial value:

$$u_1 = 3 + L1000_{a1j} \text{mod } 30 \quad (8)$$

Subsystem, a number of iterations:

$$u_2 = 3 + L1000_{a1j} \text{mod } 30 \quad (9)$$

Subsystem of b initial value:

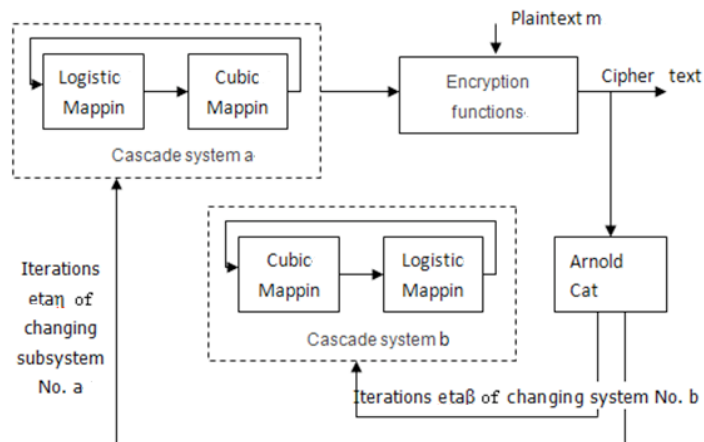


Fig. (5). Chaos encryption system block diagram.

$$b_1 = \left(\frac{r}{256} \right) \quad (10)$$

Subsystem b number of iterations:

$$y = (y_{c,i} +) \bmod l \quad (11)$$

Arnold Cat map of initial value:

$$x_{c,1} = a_1 b_1 \quad (12)$$

$$y_{c,1} = (a_1 + b_1) \bmod l \quad (13)$$

- 2) Subsystem of a and b, respectively, by the eta I (I = 1, 2, 3, %) and _i after iterations, and output the UI and wi;
- 3) UI, wi and the ith a plaintext mi after encryption function, to generate the ith a cipher text ei;
- 4) Calculate delta $\Delta = ei / 256$, and will be

$$y = (y_{c,i} + \Delta) \bmod l \quad (14)$$

$$x = x_{c,i} \quad (15)$$

As a Cubic mapping iterative initial value, after 3 times of iterative computation, xC, I + 1 and yC, I + 1;

- 5) According to xC, I + 1 and yC, I a and b + 1 calculation subsystem the next round of iterations eta I + 1 and _i + 1.

$$r'_3(i, j) = \begin{cases} 1, r_3(i, j) \geq \varepsilon \\ 0, r_3(i, j) < \varepsilon \end{cases} \quad (16)$$

$$i + 1 = 3 + L1000x_{c,i+1} \bmod 30 \quad (17)$$

Then repeat the above steps (4) ~ (17), until all plaintext encrypted.

5. THE IMPROVEMENT OF INFORMATION ENCRYPTION ALGORITHM BASED ON CHAOTIC MAPS

The text of the encryption algorithm for the maximum number of iterations of chaotic mapping limit, encrypt a plaintext character (1 byte) no more than 70 of the total of the iteration, and the famous today chaotic encryption algorithm and some improved forms, encrypt a plaintext character number of iterations required for more than 100 mostly [10].

The design process, under the premise that guarantees the security of the algorithm, to reduce the algorithm complexity, improves chose three operation simpler chaotic mapping. A cascade chaotic subsystem of Logistic mapping and Cubic the range is the same to facilitate direct cascade [10]. Mu = 2 and parameters of Logistic mapping in full chaos mapping, avoid the Logistic mapping under certain mu value is infinite the shortcomings of the window. Cascading chaotic subsystem, based on the output of the dynamic change Arnold Cat mapping iteration times Arnold Cat mapping is confirmed area, no attractor, help to improve the

area, no attractor, help to improve the security of the algorithm, and easy to implement.

In addition, the figure of the cipher text feedback is necessary. If no cipher text feedback, the repetition in plaintext characters, its cipher can present obvious regularity, make the system vulnerable to illegal attacker to decipher. In fact and cipher text feedback to make clear all affect the subsequent characters any character in the corresponding cipher text, strengthen the security of the algorithm [11].

Set image gray value as I(i, j), it meet $1 \leq i \leq M$, $1 \leq j \leq N$ isreplaced the I (i, j) in grey value (I, j). In this article, the pixel values of alternative transformation was conducted in the airspace, we designed two kinds of thinking used to implement the chaotic sequence and the pixel values of the replacement operation.

- 1) Pixel values such as formula [10]

$$I'(i, j) = \left\{ \left\{ r_1(i, j) \oplus I(i, j) \oplus r_2(i, j) + L - r_3(i, j) \right\} \bmod L \right\} \bmod 256 \quad (18)$$

Type: L said image color depth; the mod said modular arithmetic; Radius said bitwise exclusive or operation. r1, r2, r3 represents the chaotic sequence value, replace the transformation key by r1, r2, r3 provides the corresponding chaotic system, transformation can be repeated, so the encryption effect is better. Repetitions for n, together with the initial value of chaotic model and parameters as part of these key, increases the key space, increase the intensity of encryption. If the image is large, by formula (18) to see r1, r2, r3 template matrix needs to increase, thus greatly reduced the encryption efficiency. To this end, we can through the way of chunking of image is encrypted, encryption efficiency improved significantly. Decryption is encrypted inverse formula is as follows:

$$I'(i, j) = \left\{ r_1(i, j) \oplus \left[\left[I'(i, j) \oplus r_3(i, j) \right] \bmod L \right] \oplus r_2(i, j) \right\} \bmod 256 \quad (19)$$

- 2) Available pixel values transform formula

$$I'(i, j) = \bmod \left(I(i, j) + \left((r_1(i, j) * \delta + r_2(i, j) * \beta) \oplus r_3(i, j) \right), L \right) \quad (20)$$

Similarly r1, r2, r3 is chaotic maps generated respectively, according to the type (20) to know this part of the key not only contains the initial value of chaotic model and parameters, and contain an arbitrary positive integer alpha, beta, theta, so not only to achieve the effect of enlarge the key space and improve the security of the algorithm. The reverse of encryption algorithm decryption operation is through the formula.

$$I(i, j) = \bmod \left(I'(i, j) - \left((r_1(i, j) * \delta + r_2(i, j) * \beta) \oplus r_3(i, j) \right), L \right) \quad (21)$$

6. MORE CHAOS MAPPING INFORMATION ENCRYPTION ALGORITHM IMPROVED INSTANCE

This paper select one 256 x 256 grayscale image as to encrypt image, set up two keys, respectively for the two ini-

tial value of chaotic systems, one for key1 = 0.1, second is key2 = 0.2, of course, mu parameters can also be used as a key, fixed in this simulation take mu = 4. The proposed algorithm uses Matlab7.1 simulation experiment. Fig. (6) for the original image, the Fig. (7) for the encrypted image, Fig. (8) for the correct key images after decryption [11].

Step 1: Original image (clear image) by “I”, for I and M class wavelet decomposition, get C frequency coefficient matrix. Will C in low-frequency sub band coefficients of LLM and high frequency sub band coefficients HLi, LLi, HHi extract, all levels of coefficient matrix is the same size. That is to say, if the image to realize three-level wavelet decomposition, can get 10 children with matrix, a N * N image, the size of the sub band coefficients matrix is decomposed to N/8 * N/8, thus the scrambling approximation matrix can reduce the computational complexity, and the approximate coefficient has a significant effect to the visual image, the low frequency coefficient matrix P [12].

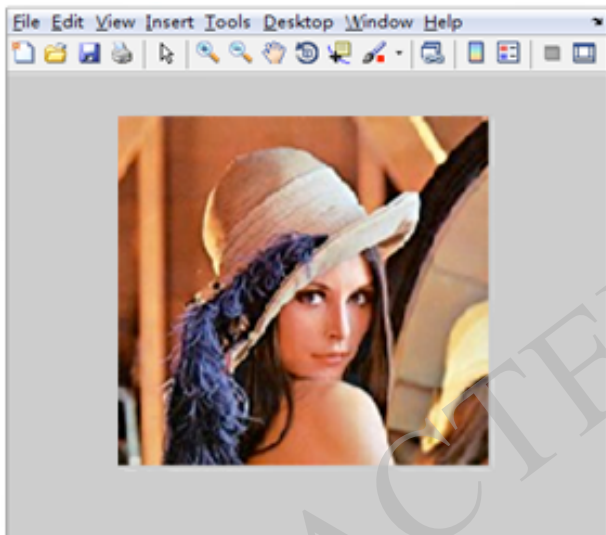


Fig. (6). Original image.

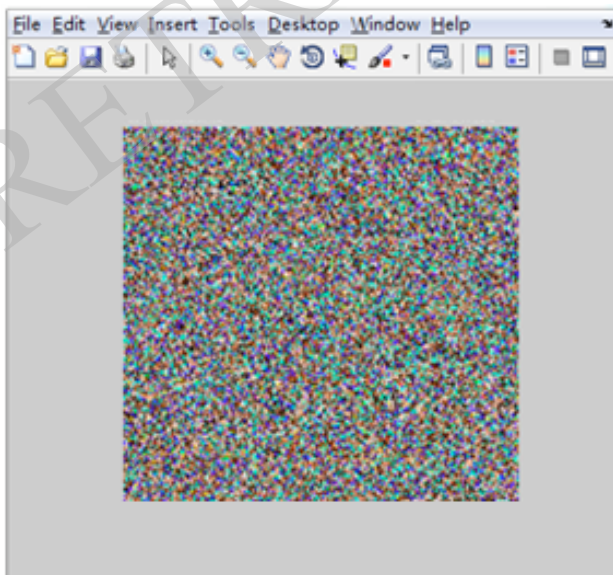


Fig. (7). Encrypted image.

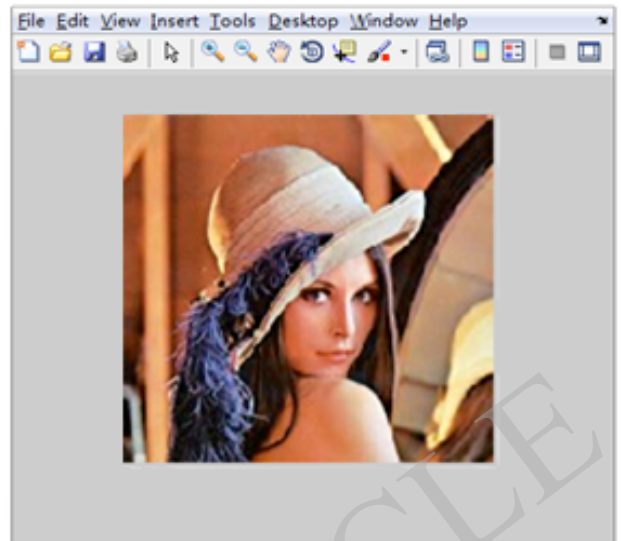


Fig. (8). Restore image.

Secret Message segmentation is divided into many pieces, each fragment cannot mean anything, just put together all of the locking plate, in order to reproduce the message out. The principle is at the sender first image data in accordance with an algorithm to divide and different people to save; and participation in the receiving end of the need to preserve the secret of man's ability to restore the original image data to be transmitted [12]. The principle of the algorithm is simple and intuitive, safe, strong anti-interference, even if the individual sub-graph leak that does not affect the entire image information leakage. However, this algorithm is a great amount of data; there are many difficulties in the process of transmission.

Step 2: To the initial value of chaotic mapping, and two-dimensional logistic mapping and parameter values, its chaotic key sequence template followed by r1, r2, and r3 and r4, said the use r1, r2, r3 and 4.3.2 method for P scrambling transformation, after scrambling matrix with P 'said. Namely,

$$P(\text{mod}((i-1)+(e+0.5), M') + 1, \text{mod}((j-1)+(f+0.5), N') + 1) \tag{22}$$

The following of e, f, and this paper discussed the value of epsilon here to set a threshold, and then through the epsilon to modify the value of r3, 0-1 sequence is obtained. Implementation process is as follows:

$$r'_3(i, j) = \begin{cases} 1, & r_3(i, j) \geq \epsilon \\ 0, & r_3(i, j) < \epsilon \end{cases} \tag{23}$$

$$e(i, j) = \begin{cases} r_2(i, j), & r'_3(i, j) = 1 \\ r_1(i, j), & r'_3(i, j) = 0 \end{cases} \tag{24}$$

$$f(i, j) = \begin{cases} r_2(i, j), & r'_3(i, j) = 1 \\ r_1(i, j), & r'_3(i, j) = 0 \end{cases} \tag{25}$$

To achieve better scrambling effect, can be used for more P in accordance with the above process several times transform. This part of the key is ey1 = (x10, x20, x30, M,

$b, k, e, n1, x_{10}, x_{20}, x_{30}$, they are the initial value. b, k for their parameters, $n1$ represents scrambling times, ϵ said setting threshold value.

Step 3: Approximate coefficient matrix P' with Step1 after wavelet decomposition in nine other detail coefficient matrix wavelet reconstruction, thus complete the initial encryption get image with I .

Step 4: In r_1, r_2, r_3 and r_4, r_4 in any of three are combined, in the process of grey value to replace them as key template. This article take r_3 and r_4 image I pixel values instead of formula (23) or (25) encrypted image I' , so for this part of the operation is completed, I' namely cipher text images. Here take the formula (23). That is:

$$I'(i, j) = \{ \{ r_3(i, j) \oplus I(i, j) \oplus r_4(i, j) + L - r_5(i, j) \} \bmod L \} \bmod 256 \quad (26)$$

$$\begin{pmatrix} x1' \\ x2' \\ \dots \\ xn' \end{pmatrix} = \begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & 2 & \dots & 2 \\ \dots & \dots & \dots & \dots \\ 1 & 2 & \dots & n \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ \dots \\ xn \end{pmatrix} \quad (27)$$

This part of the key is

key2 = $(x_{30}, x_{40}, y_{40}, k, M_1, M_2, r, n2)$. x_{30}, x_{40}, y_{40} they are the initial value. k, M_1, M_2 and rare initial value and parameters, $n2$ represents scrambling times.

Step1, Step2 can be repeated, it can greatly improve the so-called security of the algorithm, and the number of repeat can also be used as the key. Decryption process is the reverse of encryption.

CONCLUSION

This paper puts forward a composed of multiple chaotic mapping encryption algorithms. It USES cipher text feedback control map, and according to the dynamic change two cascade chaotic mapping output subsystem number of iterations, the output of the two subsystems with clear combined modulus after get the cipher text, increase the unpredictability of cipher text. Safety analysis showed that the key space is large, the algorithm sensitive to plaintext and key, can resist using differential characteristics,

resist using differential characteristics, statistical properties and phase space reconstruction of system.

CONFLICT OF INTEREST

The author confirms that this article content has no conflict of interest.

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