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Coupled Model and Algorithm of Spatial Partition Based on Multiple-Factor

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Abstract: In order to calculate the spacing and number of grid cell and get rid of data redundancy caused by subjective experience on the process of spatial partitioning, multi - factor coupled model of spatial partition was established by four constraints—Laves partition identity triangulated regular network, terrain features, data density and adaptive boundary. The model can establish adaptive boundary conditions based on discrete center and the furthest point of samples, and in order to reduce complexity and systematic errors of direct solution of terrain surface equations, the constraints of terrain features and data density were expressed by the terrain surface differential equations. The model's algorithm, based on recursive algorithm and automatic generation technology for discrete grids, can generate the 6 resolutions of grids and calculate the spacing and number of each resolution cell. Finally, when the sets of data from 110 to 440, by visualization and comparative analysis of 3 kinds of spatial partition methods, the results show that the cost of coupled model's algorithm is 1/10~1/2 of the Delaunay and Square, and also verify that the algorithm is linear convergence and can effectively solve the problem of illegal border and polygons from non – convex.

Keywords: Coupled model, data density, grid, non-convex, spatial partitioning, terrain feature.

1. INTRODUCTION

Spatial partition is the key digital technique of GIS, and an data model of GridDEM. Guo-an Tang emphasized that "it's important for spatial analysis to choose the right unit size. Oversize cell will decrease DEM 's accuracy, and the loss of large amounts of terrain information will cause image distortion; micromastia cell will cause data redundancy and decrease the display effect of macro - topography characteristics "[1]. But currently the calculation of the number and spacing of grid is based on experience, and mathematical model is scarce.

Properties of spatial partition is closely related to image reconstruction [2]. Ahuja N and Samet H held [3, 4] regular spatial partition should have two properties at least: (1) infinitely repeating pattern and apply to any scale images; (2) can be decomposed into infinitely fine pattern, and may be expressed as different grades and levels of spatial elements of arbitrary resolution. Grunbaum B and Shephard G C proved that there are only 81 equilateral partitions in regular -space [5], but restraint of the same vertices there are only 11 divisions, called Laves Division, and the Division composed of equilateral triangles, squares and hexagons identified as: $[6^3]$, $[4^4]$, $[3^6]$ (Fig. **1**).

Bell.S.B. M for further submitted other properties of spatial partition - homogenic adjacency and partition [6, 7].

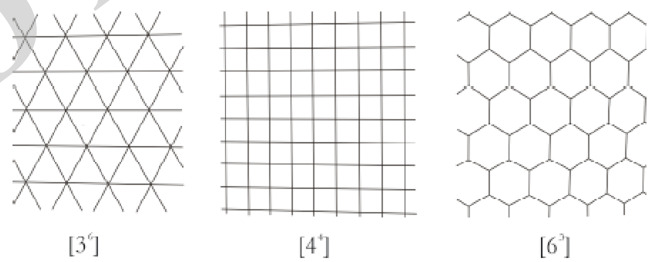

Fig. (1). Laves partition identity——$[3^6]$, $[4^4]$, $[6^3]$.

These properties are very important in image processing and drawing automatically. But in the 11 kinds of Laves Partition, only $[6^3]$ and $[4^4]$ is unconstrained regular spatial partition, and only $[4^4]$ and $[3^6]$ consistent with the properties of homogenic adjacency and the direction.

Type and spacing are the fundamental property of the Grid DEM, and they constrained by topography and density of sampled data. Hutchinson M F (2000) put forward the best spacing of Grid DEM based on mean square error of slope [8], but ignored two important factors - the data density and boundary conditions. In the areas of large terrain fluctuation, data density is relatively high, but to the terrain flat area, it's relatively low, thereby to reduce data redundancy.

The most important constraints of geo-spatial partition space are cell spacing, size, type, topography, data density, boundary conditions, and projection's type, and the model that coupled these constraint conditions in mathematical language can express geospatial division more comprehensive.

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2. MATHEMATICAL MODELING OF COUPLING SPATIAL PARTITION

Selecting grid type is restraint on properties of spatial partition, and cell's spacing depends on the precision of spatial partition and data density. Parameters of topographic relief can be expressed by the degree of slop. Establishment of the model for spatial partitioning coupling slope and data density converted solving the problems of cell spacing and amount into mathematical structure problems.

2.1. Model's Constraints and Parametric Transformation

Model need to be satisfied with the following constraints:

(1) In order to meet the requirements of properties of spatial partitioning, identified Laves Division [3[6]] as the grid's type;

(2) Precise levels-the multiple resolution, represented by a grid cell's spacing and cell's size, coupled two additional constraints with the terrain slope and density of the sample data;

(3) Boundary conditions, formed the initial grid's type by the most distant discrete points and segmentation of boundaries;

(4) Topographic factors, expressed by the differential equations of planar projection component of terrain surface;

(5) Data density, using topographic surface's differential equations to express;

(6) Spatial partitioning by Cartesian coordinate system.

2.2. Infinitesimal Element Definition of Slope

Scattered spatial data points (X,Y, Z), x is longitude (or abscissa), y is longitude (or ordinate), z is elevation. Topography is geographic continuity, and the complexity of the terrain have an important impact on the size of grid partition, thereby which affects the accuracy of DEM, and topographical constraint can be used gradient to express.

Definition of slope by differential element: the angle of point between tangent plane and horizontal plane, the value equals to the angle of point between vector of differential cell $\boldsymbol{n}$ and Z axis. Its mathematics expression:

$$\alpha = \arctan\sqrt{F_x^{\ 2}(x,y)+F_y^{\ 2}(x,y)} \tag{1}$$

F_x in an (x, y) is gradient of x axis, and $F_y(x, y)$ is gradient of y axis.

2.3. Infinitesimal Element Definition of Data's Density

Density of scattered spatial data is one of the most important indicators impacting on accuracy of grid partition. The area of topographical surface is:

$$S = \iint_{Dxy} \sqrt{1+F^2{}_x(x,y)+F^2{}_y(x,y)}dxdy \tag{2}$$

Mathematical expression of the data density is

$$\rho = \frac{N}{\iint_{Dxy} \sqrt{1+F^2{}_x(x,y)+F^2{}_y(x,y)}dxdy} \tag{3}$$

ρ represents the data density, N is the amount of discrete data.

2.4. Establishment of Mathematical Model by Coupling Factors

Slope and data density can be used differential equation of topographical surface to express. if solving the equation of topographical surface and the parameter of slope directly will bound to introduce errors. But function equivalent of slope and data density to differential equation of topographical surface can not only reduce the systematical error, also might formed mathematical model coupled 6 factors.

Plane grid is a projection in Cartesian coordinate for topographical surface:

$$S_i = M_i e_i = \frac{1}{2}F_x(x,y)F_y(x,y)\int \sin\alpha_i d\alpha_i \tag{4}$$

$\boldsymbol{S_i}$ is total area of the grid in No $\boldsymbol{i}$ area, $\boldsymbol{M_i}$ is the number of grid in No $\boldsymbol{i}$ area (i=1, 2…… 6) and $\boldsymbol{e_i}$ is the area of grid cell in No $\boldsymbol{i}$ area, α_i is gradient parameter in No $\boldsymbol{i}$ area.

The data density is further expressed by equation of topographical surface as:

$$\rho = \sum_{i=1}^{6}\frac{N_i e_i}{M_i e_i} = \frac{N_1}{M_1}+\frac{N_2}{M_2}+\cdots+\frac{N6}{M6} = \sum_{i=1}^{6}\frac{N_i e_i}{\frac{1}{2}F_x(x,y)F_y(x,y)\int \sin\alpha_i d\alpha_i} \tag{5}$$

$\boldsymbol{N_i}$ is the amount of data in No $\boldsymbol{i}$ area.

$$\sum_{i=1}^{6} N_i = N \tag{6}$$

Initial equilateral triangle grid makes subdivision infinitely, and each subdivision is divided into 4 equilateral triangles, $\boldsymbol{K_i}$ is the number of subdivision:

$$4^{K_i} e_i = S_i \tag{7}$$

Solution of the number of subdivision is:

$$K_i = [\log_4(S_i / e_i)] \tag{8}$$

$\boldsymbol{e_i}$ is the projected area of differential unit of topographical surface- $\boldsymbol{E_i}$:

$$E_i = e_i\sqrt{1+F^2{}_x(x,y)+F^2{}_y(x,y)} \tag{9}$$

The projected data density in No $\boldsymbol{i}$ area is deduced:

$$\rho_i = N_i e_i / S_i \tag{10}$$

And then:

$$M = \sum_{i=1}^{6} M_i = N_i \rho = Ni \frac{N_i e_i}{S_i} = \frac{N_i^2 i}{4^{k_i}} \quad (11)$$

M is the total number of the grids.

While in order to avoid the ambiguity of property values of grid, the constraint of cell's spacing is less than or equal to the minimum spacing of sample data:

$$l \le d \quad (12)$$

So solving the coupled model for the number and spacing of the grid is as follows:

$$\begin{cases} M = \sum_{I=1}^{6} M_i = \sum_{I=1}^{6} \frac{N_i^2 i}{4^{k_i}} \\ K_i = [\log_4(S_i / e_i)] \\ e_i = \sqrt{3}l / 8 \\ l\min \le d \end{cases} \quad (13)$$

In this Model, the constraints $l \le d$ is terminate condition of recursion.

3. ALGORITHM AND VISUALIZATION OF MODEL

Laves Division Identified [3^6] meets the requirements of properties of homogenic adjacency and the direction for spatial partitioning. Taking into issue of various resolution, the sizes of cell caused by the sparse degree of data. In particular, special constraint such as non-convex boundary, in order to solve those problems, variable scale spatial partitioning in limited area is necessary.

3.1. Algorithm

The calculation method of spacing and number for coupled model of spatial partitioning are as follows:

Step 1: Data was randomly generated in WGS84 coordinate system;

Step 2: Data was projected onto planar coordinate system, and got the farthest distance of two points, and solved the mathematical expression of circle by center point and diameter of two points. Spatial discrete points in boundaries were recorded as ***Bn (X$_n$,Y$_n$,Z$_n$), n∈N, n=1, 2, 3….*** The center ***O*** in plane coordinates was $O_x = \frac{x\max + x\min}{2}, O_y = \frac{y\max + y\min}{2}$.

Step 3: The circle divided into 6 equal scallop regions, and determined the attribution of each sample point in which scallop region, and classified.

Step 4: Solve the farthest point from the center point in each scallop, and the tangent of the furthest point t was boundary condition.

Step 5: As shown in Fig. (**2**), based on the points A,B, O, determine the slope and equations of the linear AB ;

Step 6: Based on the deflection angle of linear CF, DE relative to Line AB, find the slope and equation of line through the center O of the line CF, DE;

Step 7: Parallel lines of CD through farthest point in COD area are AB and EF, and its slope is equal to the slope of AB, so the equation of line through farthest point in COD and OEF can be obtained, by that analogy, the equation of line through farthest point can be obtained in other regions;

Step 8: Composed the line through farthest point t and lines of initial sector of equilateral triangle.

Step 9: Halve side of initial equilateral triangle until $l \le d$, and the number of attempts is equal to $\sum_{i=1}^{6} K_i = \sum_{i=1}^{6} \log_4(S_i / e_i)$, find the number of grids.

Step 9: End

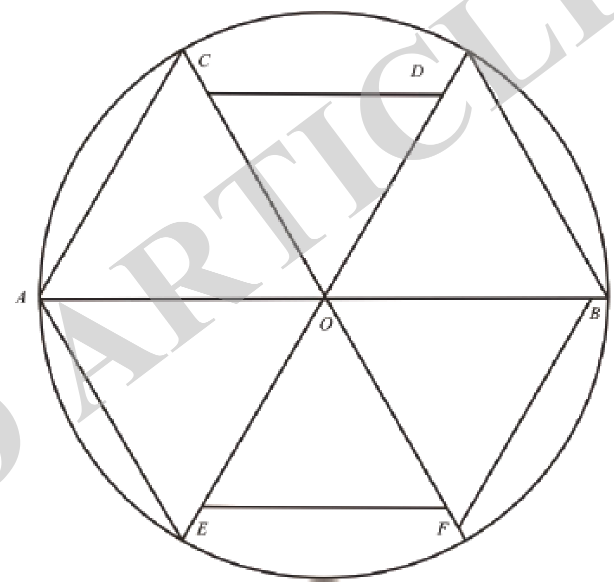

Fig. (2). Figure of the initial partition.

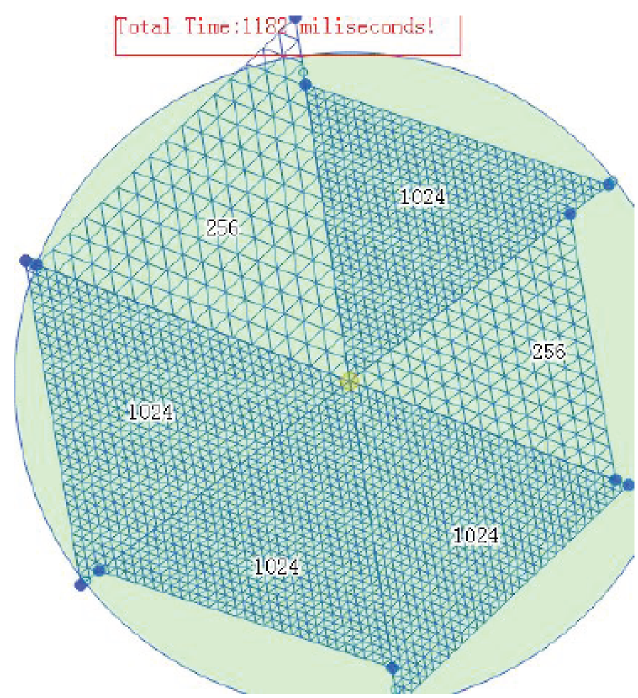

Fig. (3). Visual diagrams of 110 sets of data for partition.

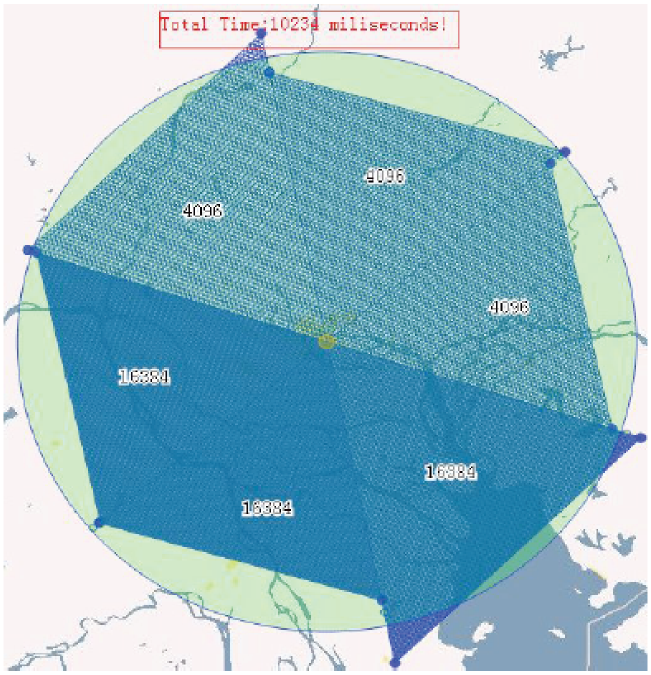

Fig. (4). Visual diagrams of 440 sets of data for partition.

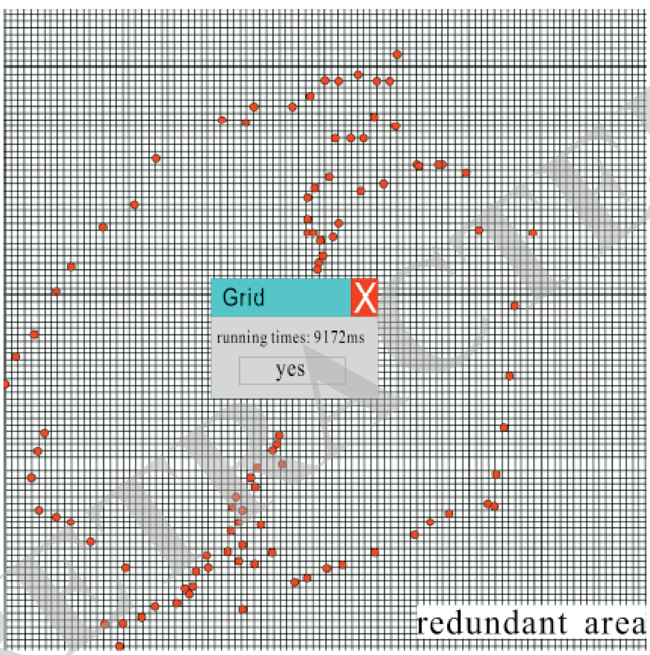

Fig. (5). Visual diagrams of 110 sets of data for partition.

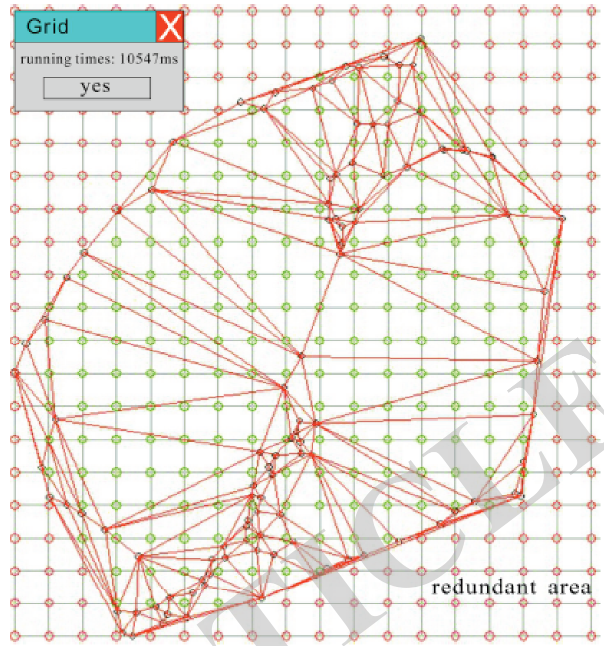

Fig. (6). Visual diagrams of 110 sets of data for partition.

3.2. Visual Verification

Test environment-CPU-AMDA8-4500M, Program language-Javascript, the browser -Googlechrome, the volume of data was from 110 to 440 sets of WGS84 datas.

From Figs. (**3** and **4**), the grids of 6 resolution in 6 regions was generated based on non-convex boundary conditions, illegal polygons and illegal boundary have also been an effective solution.

4. COMPARATIVE ANALYSIS OF MODELS

4.1. Comparative Analysis of Time's Complexity

Time's complexity T(n) and spatial complexity S(n) are the standards for measuring the quality of algorithms. In this calculation method, triangles drawn used a recursive algorithm, its smallest time's complexity was T(n) = O(logn), the best was T(n) = O(nlogn). Compared with Delaunay, which is the variable proportion spatial partition, comparative analysis of time's complexity was as shown in Table **1**:

4.2. Visualization of Comparative Analysis

4.2.1. Laves[4^4] Identifies the Division of Visualization

The test environment of Laves[4^4] was the same as the coupled model, and the result was Fig. (**5**).

Table 1. Comparative analysis table of time complexity of algorithm [9].

Model	Algorithm	Time Complexity
Delaunay	Triangle expanding method	$T(n) = O(n^2)$
Delaunay	Incremental insertion algorithm	$T(n) = O(n^2)$
Delaunay	Divide and conquer algorithm	$T(n) = O(n \log n)$
Coupled Model	Recursive algorithm	$T(n) = O(n \log n)$

Table 2. Correlation analysis table of partition under non- convex.

Method	Illegal Border	Illegal Polygon	Redundance
Coupled model	No	No	No
Laves[4^4]	Yes	Yes	Yes
Delaunay	Yes	Yes	No

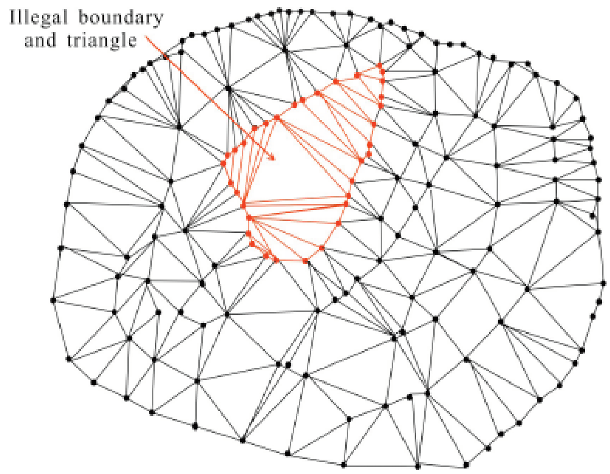

Fig. (7). Rendering of Delaunay constrained by Non-simply connected boundary.

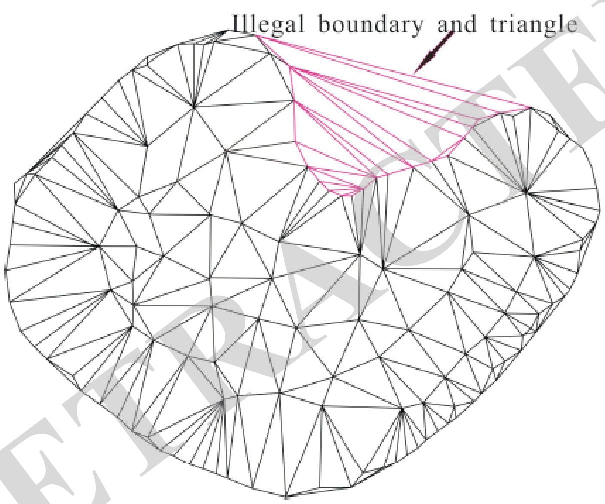

Fig. (8). Rendering of Delaunay constrained by Non- convex boundary.

4.2.2. Visualization of Delaunay Triangulation

The test environment was the same as the coupled model, and the result was Figs. (**6-8**):

4.2.3. Delaunay's Visualization by Non - Convex Constraint

Discrete non-convex sets have two cases, one is concave polygon and the other is simply connected (saddle or island). Because Delaunay was based on convex theory, Delaunay would appear illegal border and polygon by non-convex constraint, which affected the result of spatial partition.

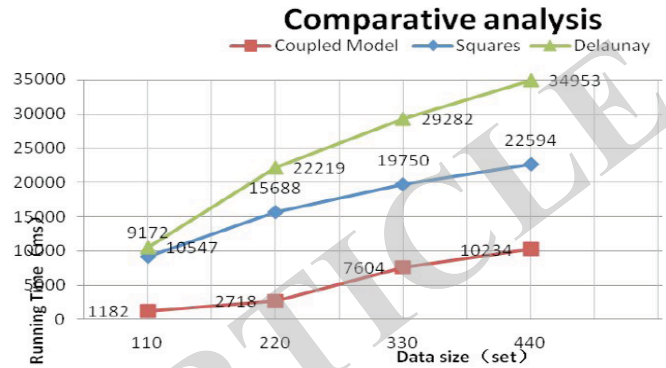

Fig. (9). Comparative analysis table of time program runs.

4.2.4. Comparative Analysis

Comparative analysis of 3 kinds of method, the cost of procedure were Fig. (**9**):

Comparative analysis of data redundancy and illegal border were Table **2**.

CONCLUSION

By mathematical modeling, the method could accurately calculate the amount and spacing of grids, and generated grid with 6 resolutions to solve the problem of illegal border, and reduced data redundancy. Besides, cost is much smaller than other methods. However, this algorithm didn't consider the effect of discrete degree of data.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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